Midterm 1, Part 1

MAT 21D, Ragone

2021 Summer II

1. Each part of this exam is **45 minutes**. At the end of the 45 minutes, you will have 5 minutes to scan & upload your exam.
2. This exam is open note, open homework, and open book. You may **not** use any external electronic or online resource, nor may you work with others.
3. There are **6 questions**, each worth **15 points**. Where a question has multiple parts, the point breakdown has been listed.
4. You must show **all** your work. Answers without work will receive no credit.
5. You may either use physical pen & paper, or an electronic note-taking app. Answer each question on a new page. Please label continuing work by the number and "cont." (e.g., if problem 3 had 3 pages, you could write "3", "3, cont.", "3, cont. 2")
6. Clearly indicate which page goes with which problem. Upload all your pages to GradeScope, and correctly match each question with the pages.
7. Stay logged in to the Zoom room until you have uploaded your exam, in case you need technical assistance. **You do not need to keep your camera on.** If you are disconnected from the Zoom room for any reason, do not worry; message me through Canvas, or send me an e-mail, and continue to work on your midterm until I respond, or until time is up.

This exam is a chance to show off what you have learned so far. If you know how to do part of a question, write down what you know. Good luck!!

Problem 1

Sketch the following regions of integrations. You do not need to evaluate the integral. Make sure you include enough labels to keep the graph clear, even if the lines are only a rough sketch.

(i) $\int_1^\infty \int_0^{1/x^2} \, dy \, dx$

(ii) $\int_0^1 \int_0^{y^2} \, dx \, dy + \int_1^2 \int_0^{\sqrt{y}} \, dx \, dy$

(iii) $\int_0^{2\pi} \int_0^2 \int_0^{r\cos\theta+2} r \, dz dr d\theta$

Problem 2

Suppose we have a cone of metal in $\mathbb{R}^3$ defined by

$$z \geq \sqrt{x^2+y^2}, \qquad 0 \leq z \leq 1$$

and suppose that the density σ of this cone at a point (x, y, z) is given by

$$\sigma(x, y, z) = x^2 + y^2$$

Compute the center of mass of this cone. Be sure to think to yourself "is this physically reasonable?".

Problem 3

The *standard simplex* is a geometric object with many applications in computer science, combinatorics, and a variety of other subjects. In 3D, a point (x, y, z) is inside the standard simplex if $x \geq 0$, $y \geq 0$, $z \geq 0$ and if $x + y + z \leq 1$.

(a) Sketch this region (clearly and well labeled).

(b) Write down an integral that computes its volume.

(c) Evaluate this integral.