Midterm 2, Part 1

MAT 21D, Ragone

2021 Summer II

1. Each part of this exam is **40 minutes**. At the end of the 40 minutes, you will have 10 minutes to scan & upload your exam.
2. This exam is open note, open homework, and open book. You may **not** use any external electronic or online resource, nor may you work with others.
3. There are **6 questions**, each worth **15 points**. Where a question has multiple parts, the point breakdown has been listed.
4. You must show **all** your work. Answers without work will receive no credit. Be specific when citing theorems.
5. You may either use physical pen & paper, or an electronic note-taking app. Answer each question on a new page. Please label continuing work by the number and "cont." (e.g., if problem 3 had 3 pages, you could write "3", "3, cont.", "3, cont. 2")
6. Clearly indicate which page goes with which problem. Upload all your pages to GradeScope, and correctly match each question with the pages.
7. Stay logged in to the Zoom room until you have uploaded your exam, in case you need technical assistance. **You do not need to keep your camera on.** If you are disconnected from the Zoom room for any reason, do not worry; message me through Canvas, or send me an e-mail, and continue to work on your midterm until I respond, or until time is up.

This exam is a chance to show off what you have learned so far. If you know how to do part of a question, write down what you know. Good luck!!

Problem 1

Let C be the oriented curve in $\mathbb{R}^2$ parameterized by $\vec{r}(t) = \alpha\cos(t)\vec{i} - \alpha\sin(t)\vec{j}$, $0 \le t < 2\pi$, and $\alpha > 0$ a positive constant.

(a) Sketch this curve.

(b) Compute the unit tangent and unit normal to this curve (choose the unit normal whose orientation agrees with our convention CCW rotation = outward normal).

(c) Now, consider the curve $-C$, the same curve with reversed orientation. Parameterize the oriented curve $-C$ (call it $\vec{r}_-(t)$) and compute the unit tangent $\vec{T}_-$ and unit normal $\vec{n}_-$ vectors along this curve. (*Note: geometric reasoning is valid reasoning!*)

Problem 2

Consider the paraboloid $f(x, y) = x^2 + y^2$.

(a) Compute and sketch the gradient vector field ∇f.

(b) Let C be the oriented curve in $\mathbb{R}^2$ parameterized by $\vec{r}(t) = 2\cos(t)\vec{i} + 2\sin(t)\vec{j}$, $0 \le t < 2\pi$.

   (i) Compute the circulation of ∇f over C. Do not use Green's theorem.

   (ii) Compute the outward flux of ∇f over C. Do not use Green's theorem.

Problem 3

Consider the vector field $\vec{F} = -y\vec{i} + x\vec{j} + z\vec{k}$. Compute the line integrals for the following oriented curves C_1, C_2:

(a) C_1 is the line starting at the point $A = (1, 1, 1)$ and ending at the origin $B = (0, 0, 0)$.

(b) C_2 is given by traveling from $B = (0, 0, 0)$ to $C = (0, 1, 1)$ along the curve $z = y^3$ in the yz plane.